

Building Models to Solve Problems

Karl A. Smith and Anthony M. Starfield

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Modeling Engineering Problems

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Modeling and problem-solving are inseparable, so much so that it is difficult to learn to solve problems without learning to model, and vice versa. We take the view that modeling is a more specific goal, that it is easier to learn to model, that it is useful to learn to model, and that incidentally one also learns a lot about problem-solving. Most problems are "solved" by constructing a representation (a mathematical expression, a graph, a manual or computer simulation program, a physical model, etc.). The process can be roughly described as (1) Formulating a model of a real system, (2) Drawing conclusions from the model, (3) Interpreting the model conclusions, and (4) Validating that the model actually works.

What is a Model?

We have asked hundreds of high school and college students and college faculty the question, "What is a model?". After giving them a moment to reflect, jot down their ideas and perhaps share them with a partner, we randomly select students to answer and we record the answers on a transparency.

Students' answers from a recent class of third-year engineering students included:

- a representative system which simulates a situation
- a graphical representation
- a simplification--physical or conceptual
- a decision-making tool
- a means of testing a situation
- establishes parameters in which to work

Our role during this period is to guide the discussion, dignify all the students' contributions, and build on their understanding. Though they usually mention the main points about representation,

simplification, abstraction, assumption, notation, algorithm, system, and generalization, one central feature of a model that the students seldom mention is purpose. We stress that one cannot build a model without understanding the purpose. When pressed, students' responses to the question of purpose typically include: to predict, to explain, to communicate, to understand, to control, to help decide, and to learn.

Through building models our students learn to look at problems in terms of general patterns, then slowly refine their approach to attain a better model instead of simply asking "What is the answer?" or using the typical problem-solving approach "What am I given? What am I asked for? What rule or formula can I recall that connects the two?". Information gleaned by asking students to reflect on their experience indicates students learn a great deal about problem solving by building models. Sara's experience in an ecological modeling course is representative:

I think the biggest revelation I have had in this course so far is a very basic one, but it's difficult to describe. It's a new approach to problems, and a new way to use computers. I tend to have a brute-force approach to complicated problems--I begin at some entry point and slowly work through to an outcome. Then I begin again and make some different choices. The result is an array of outcomes, unless I have forgotten some possible choices. What I have learned is to collect the rules of the system, and build a picture of the system using those rules. Then it is easy to try different choices and view the outcomes, and more importantly it is easy to review the rules and change them or add to them. And it is easier to see which rules matter most, and which are less important. The difference between this approach and the old one is that I can more easily focus on the rules to the system, and on the problem itself. The mechanics are easy because they are built in, where before I took this course I tended to become so entangled in the mechanics that I couldn't see the system clearly.

We stress that learning in all disciplines involves constructing models, investigating ideas and developing problem-solving skills. These activities are not limited to students in science and mathematics. They are shared by all who have a desire to understand, to interpret and to explain. Business managers use market forecasting models, resource allocation models, and decision-making models. Airline pilots train in flight simulators, models of real aircraft cockpits. Weather forecasters construct and use very complicated (and mathematical) atmospheric models to keep us informed about the (near) future. Teachers have models of their students--"empty vessel to be filled," "blank slate," "product to be processed on an assembly line," "mind to be developed," or "person to work with." Medical doctors use absorption models in prescribing drug dosages, trace-element accumulation models for detecting lead poisoning, and cholesterol-level models for predicting heart attack risk.

Modeling means constructing a simplified representation of some physical, biological or social phenomenon which is too complicated to represent in all the details of its entirety. Students manipulate their model to gain understanding of the phenomenon being modeled. We offer advice along the way, including

1. Keep it simple. Beware of building a complicated model when a simple one will do.
2. Try to understand the problem. Imagine that you're in the situation--what does it look and feel like? how are things changing?
3. The principal benefit of modeling is often associated with what is learned while trying to build the model.
4. GIGO: Garbage-In-Garbage-Out. A model is not any better than the information that goes into it.

Working Cooperatively to Model and Solve Problems

First year college students in Starfield & Smith's **How to Model It** course are organized into groups of three within the first 10 minutes of their first class. This arrangement surprises some of the students--however; most of them eagerly find the other members of their group and introduce themselves. A task such as the ping-pong problem is assigned and one copy is given to each group. Instructions to the students include: each group is to formulate one answer to present, make sure everyone participates, and make sure everyone can explain your group's answer.

While the students are working on the problem, we circulate among the groups; listening as the students discuss the problem with each other, occasionally intervening to ask a student to explain, and continually providing support and encouragement for the group work.

After we call the whole group back together, individual students are randomly selected (by asking each student to choose a number between 0 and 9 and using a 10-sided die to generate a random number). These randomly selected students give their group's answer and explain the method used to arrive at their answer. Several answers are requested and recorded on an overhead. Answers and methods are compared in terms of the formulation and assumptions. The usefulness of formulating an algorithm and using a notation system are discussed.

The final step in the cooperative groups involves the students processing their work in two ways. First they discuss how well they solved the problem, including their use of strategy and how each member feels about the group's answer. Second, they process how well they worked as a group--what things went well and what things they need to work on to function more effectively together.

The essence of this classroom structure is students working together to get the job done. Our book, *Active Learning: Cooperation in the College Classroom*, carefully defines cooperative learning, describes how to do it, and provides detailed rationale.

Designing, Testing and Refining Models

Our modeling courses concentrate on problem formulation, setting up models, and drawing conclusions from the models. Students work in small, cooperative groups on a number of problems using a spreadsheet tool (or a more general modeling tool such as **Mathcad**) run on a personal computer. Students construct mathematical and computer models for each problem and then manipulate their model by varying its parameters and recording the effects to gain understanding of the physical, biological or social phenomenon being represented. Interesting problems are used to help students discover how to make assumptions, build a model and interpret their results. The problems presented to the students require formulation, solution, discussion, iteration and new solutions. They are problems which involve engineering, physics, biology, geometry and diverse others, including classics such as "How many ping-pong balls could you fit in this room?"--an example of the type of problem we set in an introductory course for first-year students (See Figure 1).

Insert Figure 1 about here

1. **Building a Quick Model: A One-Minute Answer**

Let's take a detailed look at the ping-pong problem introduced above. The classroom activity involves a quick, one-minute estimate by each individual followed by five-minute estimate by trios. We ask the students to look around the room and take just 60 seconds answering the question: "How many ping-pong balls could you fit into the room?". The students are momentarily bewildered, but they soon start looking around the room, then at the size of the ping-pong ball, and being doing either mental or paper-and-pencil calculations. Recent answers from a group of 17 high school students were:

10 billion; 12,000; 1 million; 500,000; 5 million; 100 million; 10 million; 300 million; 100 million; and 12 million

Not everyone gives an answer, particularly a numerical answer. Some give the answer, "a lot." Although these one-minute problems are difficult for students at first, we persist in posing them because we are determined to improve students' estimation skills.

2. Explaining Your Model: A Five-Minute Answer

The students are next asked to work with their group for five minutes to answer the same question, 'How many ping-pong balls could you fit into this room?' AND to develop an explanation of how they arrived at their answer.

Students quickly formulate an approach and often divide the task--one student measuring the room, another getting a calculator, and the third measuring the ball. Students are intensely focussed during this time. We circulate among the groups, eavesdropping on their conversations, stopping to contribute or clarify, respond to questions (often by turning the question back or suggesting that they make an assumption and go on). When five minutes has elapsed, we call for the answer from each group.

Five-minute answers from each of the six groups of high school students mentioned above were:
20 million; 17 million; 15 million; 5 million; 27 million; 27 million.

Students are asked to compare the one-minute and the five-minute answers which are all displayed on the board or overhead. They often note, jokingly, that there are fewer five-minute answers and that every group obtained an answer. They usually observe that the spread is much less for the five-minute answers.

Since there is a range from 5 million to 27 million, we go on to explore their models to see if we can understand the differences in their estimates. Assumptions are usually considered first. Assumptions listed by the high school group were: room is an empty box, room is a rectangular box, no furniture or

people present, ignore room irregularities, ping-pong ball is a cube, and furniture may account for some of the packing.

3. Presenting and Discussing the Alternatives: How Does Your Model Work?

We ask the groups, "How did you get it--did you construct a model? *If so, describe your model--*" Each group's model is recorded on a overhead transparency. Different groups formulate this problem in different ways and the range of answers is often due to their initial, often unspecified assumptions. For instance, some groups estimate how many balls would fit on the wall at one end of the room, and then multiply by the number that would fit along the length of the room. Each of the high school students' groups estimated the volume of a ping-pong ball and the volume of the room, and divided the one into the other.

The group that got the answer "5 million" said:

let L be the length of the room, 28 feet,

let W be its width, 35 feet,

let H be its height, 10 feet,

and let D be the diameter of a ping-pong ball, 1.5 inches.

$$V_{room} = LWH$$

Then the volume of the room is

$$V_{ball} = D^3$$

and the volume of a ball (treating it as a cube) is

so number of balls equals

$$N_{balls} = \frac{V_{room}}{V_{ball}} = \frac{LWH}{D^3}$$

One of the groups that answered "27 million" volunteered that they used basically the same

$$V_{ball} = 4\pi \frac{r^3}{3}$$

approach, except they used the volume of a sphere for the ping-pong ball.

We asked the students to explore the difference in the estimates due to the assumptions about the volume occupied by the ping-pong ball. They said treating the ball as a sphere would give an upper limit (since it assumes no space between the balls) and treating it as a cube would give a lower limit. Several

$$\frac{N_{balls(upperlimit)}}{N_{balls(lowerlimit)}} = \frac{\frac{3V_{room}}{4\pi r^3}}{\frac{V_{room}}{(2r)^3}} = \frac{6}{\pi} \approx 2$$

students quickly constructed the ratio of upper limit over lower limit by dividing the two expressions

4. Deciding on a Model: How Good an Answer Do You Need?

Since the range of students' answers is from 5 to 27 million and assumptions about the ping-pong ball (cube or sphere) only make a difference of about 2 times, each group was asked to refine their model.

At this stage we typically try to establish the method that would give the best possible answer. We ask "How would you arrive at the best answer you can give to the question, 'How many ping-pong balls could fit in this room?'" They think about this question and discuss it with their group before answering. One group recommends, "Measure the room and ball more accurately." Another group responds "Fill a one-cubic foot box with ping-pong balls and count them; then multiply by the volume of

the room." Someone in the back says "Fill the room up with ping-pong balls and count them!" We ask if anyone can think of an approach that will give a better answer than that? After a few challenges, there is usually agreement that filling the room and counting will give the best possible answer.

As frustration rises, someone inevitably asks "Who cares how many ping pong balls you can fit in this room." We respond "Thanks for asking the most important question!" How good an answer do you need? Is it worth the effort? How accurate an answer on the number of ping-pong balls you are willing to accept depends on how good an answer you *need*. Principally, it depends on the **purpose** of the model. You cannot really answer the question, "How many ping-pong balls could fit in this room?" unless you are told how good an answer is needed.

5. Putting it in Writing: Building a "final" model

For the final part of this activity each group was asked to formulate a two-hour answer and submit it in written report form. We stressed that their report address the question "What is your model most sensitive to?" Each group implemented their model on a spreadsheet; systematically varied the dimensions of the room, the dimension of the ball, and the packing; clarified and tested their assumptions; and reported their findings. The following report from David, Mark, and Ernie is typical of first-year college students' two-hour answer.

In order to arrive at this final answer, several assumptions had to be made. The radius of the ping-pong ball was determined by measuring the circumference at 4-11/16 inches and dividing this by 2π . The "2-dimensional packing" is allowing for adjacent balls filling in a portion of the space between balls--in fact this is an "effective radius", measuring half of the distance (along the horizontal component) between centers of adjacent balls. The same holds true for "3-dimensional" packing", with the radius being derived from a formula for the height of a tetrahedron (assuming that the vertices of the tetrahedron are at the centers of adjacent balls). This radius is then used as one-half the length of one edge

of a cube, the "effective volume" of a ping-pong ball being the volume of that cube. The "trapezoidal area" is not entirely accurate, as it assumes that both the front wall and the straight line connecting the two vertices of the back wall are parallel, and they are not. The "area of remaining arc" was approximated with an isosceles triangle rather than an arc, with the error estimated at less than 1000 in². The column area assumes end walls perpendicular to side walls. The total number of balls is arrived at by dividing the total room volume by the "cubic volume" allowing for "3-dimensional packing". In our estimates, the margin for error in the result should be less than one million ping-pong balls. This could be improved upon given more time and more accurate room measurements. It is interesting to note how close the "effective volume" of a ping-pong is to its actual spherical volume. Most of the volume between balls is filled when suitable allowance is made for "packing". In higher dimensions, this "effective volume" would probably become even closer to the actual volume.

Further refinement of the model for the pong-pong ball problem is available, along with models for many additional problems, in our books *How to Model It* and *Building Models for Conservation and Wildlife Management*.

What Students Learn

Since reflection is an integral part of the process of building models to solve problems with students, we periodically ask students to step back and reflect, "What did *you* learn about modeling from this exercise?"

As the students contribute their insights, we typically look for and refine the following points:

1. Both the one-minute and five-minute exercises illustrate the point that a model is a partial

rather than a complete representation.

2. Even a very rough answer is better than no answer at all. We encourage student to come up with the best answer within the available resources. Often a range (the answer is between --and --.) is better than a single number.
3. A model that is inadequate under one set of circumstances may be the best that you can do under another set of circumstances. It follows that the design of a model depends as much on circumstances and constraints (of money, time, data or personnel) as it does on the problem that is being solved. It also follows that the assumptions one makes depend on the circumstances in which one solves the problem.
4. A symbolic representation (choosing a notation and building a formula or formulae) is 'clean' and powerful. It communicates, simply and clearly, what the modeler believes is important, what information is needed and how that information will be used.
5. Sometimes one uses models implicitly (without being aware that one is doing so); at other times one consciously or explicitly constructs or uses a model. An **explicit** model is an indispensable tool for solving problems and for talking about the solution.

Students have ample experience solving problems that have a unique answer, in other words, problems that have an answer--**the answer**--printed at the back of the book. Although these problems have a certain limited usefulness in the improvement of students' mechanical problem-solving skills, they are not appropriate for developing students' modeling abilities. The appropriate problems to assign for the development of students' modeling skills do not have a simple unique answer--their answers depend on the problem's formulations and assumptions.

Problems in the real world do not magically appear in a form ready to be solved. They are messy and often not clearly identified, or if identified, the label or identification may be incorrect or misleading. The principal problem is often figuring out what the problem is. In short, real problems (in contrast to

text-book problems) are not naturally well formulated. Even after identifying the problem, much iteration is usually required to create a satisfactory solution. Students, however, often think that once they have *solved* the problem, that is, generated an answer, they are finished. And for them, the sooner the better-- they won't reconsider their work unless forced to. The attitude engendered in students by problems with a single answer doesn't prepare them for tackling and solving the problems they will encounter in the world.

For several years we've been conducting a modeling course for upper division and graduate students in ecology, in which these students are teamed with first-year engineering, science, and math students. Interdisciplinary teams work together to formulate and solve the problems. During the process of building models together, the engineering students learn some biology and the ecology students learn some mathematics and computing. Kevin, a first-year engineering student, said of the modeling in an ecology course:

The last concept found to be very important is the analysis of the model upon completion.

To merely use the answer received in a model as a justifiable answer is completely contradictory because you have not used the model as a means of justifying your answer but rather as a tool that is assumed to produce correct answers. When one arrives at an answer, the structure of the model and its resolution and sensitivity are used to justify the answer that one arrives at. By analyzing your model and questioning the logic behind some of the subroutines, the modeler begins to further define the scope of the question and is thereby able to create a better model. Also in the process the modeler learns more about how changes within the model may have an effect on the final outcome of the model. Sometimes by just analyzing the structure of the model one can fairly accurately predict its outcome without even using values within the model. A good example of this type of analysis was the mathematical model predicting zebra and water buffalo growth

in a preserve.

Sara, an ecology graduate student whose comments opened this chapter, continues:

. . .this approach to models is the most important thing I have learned this quarter, in any of my classes. I am good at organizing things logically, but for several years I have been frustrated; the problems I want to be able to think about have grown too large and too complicated for my brain to be able to handle all of the interacting facets at once. I want to be thinking about the relevance of various factors and their interactions, but I end up struggling so hard just to keep them all straight that I can't also think about how they interact. Your course has shown me the obvious--how to store the simplest version of the interactions in a system so I don't have to worry about mechanics and my brain has room to think about the system itself--how it works, which parts matter most, what the effects of the parts I haven't added yet are, and what the implications of the whole thing are under various conditions. It sounds maudlin, but this has been a real revelation to me.

Our students' comments on our modeling courses indicate they learn much more than to ask the question, "Do I have the right answer?"

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